The InApprox Method - A New Algorithm for Computing the Weight Set Decomposition for Tri-Objective Mixed-Integer Problems

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Mots-clés : multi objective optimization, multicriteria optimization, weight set decomposition, mixed-integer, scalarization.

The Weighted Sum Method is a well-known and widely used scalarization method in multi objective optimization. In a non-interactive context one is often interested in finding all so-called (extreme) supported solutions and/or their images. This is closely related to the task of finding a weight set decomposition, that is decomposing the set of all positive weight parameters into subsets for which each of these extreme supported images are optimal. While this is easy to achieve for two objective functions by Dichotomic Search, however for three or more objectives this task is challenging.

In recent years, finding all extreme supported nondominated solutions or images has been in the focus of research : The first contribution was achieved by Przybylski, Gandibleux et Ehrgott [4], where they gave first theoretical results and presented an algorithm for tri-objective mixed-integer problems (and recursively for more than three objectives). Özpeynirci et Köksalan [3] presented an algorithm for multi objective integer problems, while Alves et Costa [1] focused on an interactive use for their algorithm. Recently, Bökler et Mutzel [2] proposed the first output sensitive algorithm for combinatorial problems. An algorithm is output sensitive, if it has a polynomial running time in the encoding length of both in- and output.

In this talk, we present a new algorithm for finding the weight set decomposition for tri-objective mixed-integer problems. Our algorithm makes use of the results in [4] and also we propose new theoretical results regarding the facets of the weight set component. These separation theorems clarify, if a line between several points in the weight space is indeed an edge (facet) or sub-edge of some weight set component.

The crucial innovation of our algorithm is that the original problem is iteratively transformed into a bi-objective problem by a weight matrix that is solved via Dichotomic Search. The algorithm starts by exploring the boundary of the weight set with such bi-objective problems. In the next step, a convex hull algorithm finds a subset of the component of each known extreme supported nondominated image. Such a subset is expanded as follows : Given a current non-definitive facet of the subset, search the line orthogonal to this facet by solving a bi-objective problem like above. Then, using the separation results, the algorithm checks, if it has found a (sub-)edge of the weight set component, which can be extended by another search. This time, the search with the bi objective problem is done along the sub-edge. The algorithm terminates, as soon as all components have definitive edges only and hence are fully explored.

The correctness of our algorithm is proven using the properties of the weight set components and the separation results. Further, we are able to present a running time : Our algorithm returns a weight set decomposition for tri-objective mixed-integer problems in \( O(|Y_{ESN}|^2 \cdot T_{WS}(n, m)) \), where \( Y_{ESN} \) is the set of extreme supported nondominated images and \( T_{WS}(n, m) \)
is the running time of a weight sum algorithm. Similar to the algorithm of Bökl er et Mutzel [2], this algorithm is output sensitive, if the weighted sum algorithm is output sensitive.

The algorithm has been implemented in Julia using the vOPT Solver and we provide both a numerical example and a computational study. We close our talk by a report on our experiences with Julia and the vOPT Solver and highlight how a generalization of our algorithm for more than three objectives can be achieved.

Références


